Directions: Always use proper notation! (i.e. for intervals, points, functions, etc.) Unless told otherwise, **all** numbers in answers should be as ***common fractions*** in *reduced* form. (You may also use decimal form if the value is terminating or proper notation is used to denote the repeating digit sequence.)

1) A doctor orders a medicine dosage that is A_t milliliters (mL) of a P_t% solution. A nurse has both a P_a% solution and an P_b% solution available in this medicine, but not P_t%. How many milliliters of each available solution should be mixed together to produce the A_t mL dose of P_t% solution?

KEY **Time: 10 minutes** **Calculator: OK** **KEY**

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Setting up the chart for what's given I see that:

Solution	Concentration	Amount	Volume
$P_a\%$	$\frac{P_a}{100}$	A_a	$\frac{P_a}{100}A_a$
$P_b\%$	$\frac{P_b}{100}$	$A_t - A_a$	$\frac{P_b}{100}(A_t - A_a)$
$P_t\%$	$\frac{P_t}{100}$	A_t	$\frac{P_t}{100}A_t$

I set up my equation by adding the first two entries in the last column and setting their total equal to the last line's last column entry:

$$\frac{P_a}{100}A_a + \frac{P_b}{100}(A_t - A_a) = \frac{P_t}{100}A_t$$

I can then multiply both sides by 100 to rid myself of the fractions before I proceed to solve:

$$\begin{aligned}
100\left(\frac{P_a}{100}A_a + \frac{P_b}{100}(A_t - A_a)\right) &= 100\left(\frac{P_t}{100}\right)A_t \\
P_aA_a + P_b(A_t - A_a) &= P_tA_t \\
P_aA_a + P_bA_t - P_bA_a &= P_tA_t \\
(P_a - P_b)A_a + P_bA_t &= P_tA_t \\
-P_bA_t & \quad -P_bA_t \\
(P_a - P_b)A_a &= (P_t - P_b)A_t \\
\div(P_a - P_b) & \quad \div(P_a - P_b) \\
A_a &= \frac{P_t - P_b}{P_a - P_b}A_t
\end{aligned}$$

Now back-substituting into the relationship between the two available solutions I find that $A_t - \frac{P_t - P_b}{P_a - P_b}A_t = \frac{P_a - P_t}{P_a - P_b}A_t$.

So the nurse needs $\boldsymbol{\frac{P_t - P_b}{P_a - P_b}A_t}$ **mL of the $\boldsymbol{P_a}$% solution** and $\boldsymbol{\frac{P_a - P_t}{P_a - P_b}A_t}$ **mL of the $\boldsymbol{P_b}$% solution** to create A_t mL of a P_t% solution.

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$P_t\%$	$\frac{P_t}{100}$	A_t	$\frac{P_t}{100}A_t$

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Now back-substituting into the relationship between the two available solutions I find that $A_t - \frac{P_t - P_b}{P_a - P_b}A_t = \frac{P_a - P_t}{P_a - P_b}A_t$.

So the nurse needs $\boldsymbol{\frac{P_t - P_b}{P_a - P_b}A_t}$ **mL of the P_a% solution** and $\boldsymbol{\frac{P_a - P_t}{P_a - P_b}A_t}$ **mL of the P_b% solution** to create A_t mL of a P_t% solution.

Checking our solution against the previous problem which had actual numbers in it, we see that: $A_t = 25$, $P_t = 50$, $P_a = 30$, and $P_b = 80$. This gives us the need for $\left(\frac{50-80}{30-80}\right)25 = \left(\frac{3}{5}\right)25 = 15$ mL of the 30% solution and $\left(\frac{30-50}{30-80}\right)25 = \left(\frac{2}{5}\right)25 = 10$ mL of the 80% solution — just as before!

KEY **KEY**