Directions: For multiple choice (A, B, C, etc.), **circle** *the letters* of all correct choices. For TRUE/FALSE, **circle** *the word* TRUE or *the word* FALSE. Otherwise follow the directions given.

Unless told otherwise, **all** numbers in answers should be as ***common fractions*** in *reduced* form or in radical form similarly reduced. (You may also use decimal form if the value is terminating or proper notation is used to denote the repeating digit sequence.)

Work easier problems first. Write out your plan for harder problems before beginning them. Note that the last two pages are bonus!

1) Construct a truth table for this compound proposition: $(p \to q) \wedge (\neg p \to q)$.

What is the proposition equivalent to?

A) p B) q C) $p \wedge q$ D) $p \vee q$

2) Use De Morgan's laws to find the negation of each of the following statements.

i) Jan is rich and happy.

ii) Carlos will bicycle or run tomorrow.

3) Determine the truth value of each of the following statements if the domain for all variables consists of all integers.

i) $\forall n \exists m(n + m = 0)$

ii) $\exists n \exists m(n + m = 4 \wedge n - m = 1)$

iii) $\exists n \exists m(n^2 + m^2 = 5)$

iv) $\exists n \forall m(n < m^2)$

4) Prove that there is a positive integer that equals the sum of the positive integers not exceeding it. Is your proof constructive or nonconstructive?

5) Evaluate the following implications.

   TRUE / FALSE If $1+1=2$, then $2+2=5$.

   TRUE / FALSE If $1+1=3$, then $2+2=4$.

   TRUE / FALSE If $1+1=3$, then $2+2=5$.

6) Use the contrapositive approach (indirect proof) to show that for every positive real number r, if r is irrational, then $\sqrt{r}$ is also irrational.

7) Prove that if x and y are real numbers, then $\max(x,y)+\min(x,y)=x+y$. [Hint: Use a proof by cases, with the two cases corresponding to $x \geqslant y$ and $x < y$, respectively.]

8) Use a proof by contradiction to prove that for $a, b,$ and c integers with $a \neq 0$ that if $a \not| bc$, then $a \not| b$.

9) Use a direct proof to show that the sum of two odd integers is even.

10) You are given this fact: "If $n > 2$ is a prime, then n is odd." Then you are asked to prove this proposition: "If p is a prime with $p > 2$, then $p+1$ is not prime." [Note that this proposition is *for all* primes greater than 2.]

Bonus Problems:

11) State what the following statement means when the *or* is inclusive vs. when it is exclusive. Which type of *or* do you think was intended?

Dinner for two includes two items from column A or three items from column B.

12) Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.

i) $\exists x \forall y (xy = y)$

ii) $\forall x \forall y \exists z (x + y = z)$

13) Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not equivalent. (And thus that implication is not associative.)

14) Prove the given fact from #10: "If $n > 2$ is prime, then n is odd." [Hint: The premise could have also been written as "$n > 2$ and n is prime".] [Hint 2: An indirect approach might be easiest.]

15) Prove that if m and n are integers and mn is even, then either m is even or n is even.

16) Express the following English sentence using predicates, quantifiers, and logical operations.

Each participant on the conference call whom the host of the call did not put on a special list was billed.

What is the opposite of the sentence? (It may help to negate your symbolic version first.)