3.5.6 1a)

$$P(n) : \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Base Case $P(1)$:

LHS: $\sum_{i=1}^{1} i^2 = 1^2 = 1$

RHS: $\frac{1(1+1)(2\cdot 1+1)}{6} = \frac{1\cdot 2\cdot 3}{6} = \frac{6}{6} = 1$

Inductive Hypothesis Assume that for $k \geq 1$ that $P(k) : \sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$ is true.

Goal State $P(k+1)$:

$$\begin{aligned}\sum_{i=1}^{k+1} i^2 &= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6} \\ &= \frac{(k+1)(k+2)(2k+3)}{6}\end{aligned}$$

Inductive Step

$$\begin{aligned}\sum_{i=1}^{k+1} i^2 &= \sum_{i=1}^{k} i^2 + (k+1)^2 \\ &= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \\ &= (k(2k+1) + 6(k+1))\frac{k+1}{6} \\ &= (2k^2 + k + 6k + 6)\frac{k+1}{6} \\ &= (2k^2 + 7k + 6)\frac{k+1}{6} \\ &= (2k^2 + 4k + 3k + 6)\frac{k+1}{6} \\ &= (2k(k+2) + 3(k+2))\frac{k+1}{6} \\ &= (2k+3)(k+2)\frac{k+1}{6}\end{aligned}$$

Since this worked in the base case and P(k) implied P(k+1) we have proven the general case P(n).